Decoupling of High-Gain Multivariable Tracking Systems

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The problem of achieving a control system design that produces the tracking of command inputs and the decoupling of outputs of high-gain multivariable control systems is considered. The tracking requirements and the conditions under which the decoupling of outputs is possible are given. A procedure for selecting the design parameters is described. A new synthesis procedure for decoupling is described. The procedure is illustrated through numerical examples for several fighter aircraft performing a number of maneuvers.

I. Introduction

OUTPUT decoupling is an essential requirement in the design of many multiple-input, multiple-output (MIMO) control systems. The design of high-gain error-actuated MIMO tracking systems that follow a constant command input vector is presented in the recent literature.[1,2] For controllable and observable systems represented by state and output equations having the forms

$$\dot{x}=Ax+Bu \tag{1}$$

$$y=Cx \tag{2}$$

an essential requirement of output feedback design is that the matrix product CB must have full rank. However, when CB does not have full rank, it has been shown[3] that the introduction of extra output measurements can lead to the design of a controller which achieves tracking. The system also rejects[1] a constant disturbance. The necessary and sufficient conditions for output decoupling are defined for state feedback and output feedback systems.[4-7] The application of those conditions is extended in this paper to systems for which CB is rank deficient. Thus, the purpose of this paper is to present a comprehensive design method for obtaining an appropriate transducer matrix M that not only yields the required tracking, but also minimizes the coupling between outputs. This is accomplished by computing a transducer or measurement matrix such that the asymptotic structure of the transfer function matrix is diagonal.

The paper is organized as follows: tracker theory is briefly described in Sec. II for completeness. The details can be found in Refs. 2 and 3. The main contribution of this paper is the synthesis of the decoupling given in Sec. III, which is demonstrated with the help of two examples in Sec. IV. Conclusions are given in Sec. V.

II. Tracker Theory

A completely controllable linear time invariant system may be represented (by a transformation of states, if necessary) by the state and output equations of the respective forms

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u(t) \tag{3}$$

and

$$y(t) = [C_1 \;\; C_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{4}$$

where B_2 is a square, nonsingular matrix. When $CB=C_2B_2$ is rank deficient, new output measurements $w(t)$ given by Eq. (5) are selected to achieve tracking,

$$\begin{aligned} w(t) &= y(t)+M\dot{x}_1(t) = [C_1+MA_{11} \;\; C_2+MA_{12}] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\ &= [F_1 \;\; F_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{aligned} \tag{5}$$

where M is an extra plant measurement matrix that must be designed so that $FB=F_2B_2$ has full rank and decoupling is achieved. With constant inputs $v(t)$, the steady-state values of the states for the designed closed-loop system with the proper controller are also constants and, therefore, from Eq. (3),

$$\lim_{t\to\infty} \dot{x}_1(t) = \lim_{t\to\infty} [A_{11}x_1(t)+A_{12}x_2(t)] = 0 \tag{6}$$

Thus, for arbitrary initial conditions, the error vector $e(t)=v(t)-w(t)$ assumes[3] the steady-state value

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} \{v(t)-w(t)\} = \lim_{t\to\infty} \{v(t)-y(t)\} = 0 \tag{7}$$

Since the state equations for $\dot{x}_1(t)$ contain no inputs, the steady-state value of $w(t)$ is equal to the steady-state value of $y(t)$, and thus tracking is achieved.

The high-gain error-actuated controller is governed by the proportional plus integral control law of the form

$$u(t)=g[K_0e(t)+K_1z(t)] \tag{8}$$

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where g is a scalar gain parameter, K_0 and K_1 controller gain matrices, $e(t)=v(t)-w(t)$ the error vector between constant command inputs $v(t)$ and output measurements $w(t)$, and $z(t)$ the time integral of the error $e(t)$. A functional block diagram of the complete tracker is shown in Fig. 1. The error states are governed by the differential equation

$$\dot{z}(t)=e(t)=v(t)-w(t)=v(t)-F_1x_1(t)-F_2x_2(t) \qquad (9)$$

In Eqs. (1-9), $x_1(t)\in\Re^{(n-\ell)}$, $x_2(t)\in\Re^{\ell}$, $u(t)\in\Re^{\ell}$, $y(t)\in\Re^{\ell}$, $w(t)\in\Re^{\ell}$, $A_{11}\in\Re^{(n-\ell)\times(n-\ell)}$, $A_{12}\in\Re^{(n-\ell)\times\ell}$, $A_{21}\in\Re^{\ell\times(n-\ell)}$, $A_{22}\in\Re^{\ell\times\ell}$, $B_2\in\Re^{\ell\times\ell}$, $C_1\in\Re^{\ell\times(n-\ell)}$, $C_2\in\Re^{\ell\times\ell}$, $F_1\in\Re^{\ell\times(n-\ell)}$, $F_2\in\Re^{\ell\times\ell}$, $e(t)\in\Re^{\ell}$, $v(t)\in\Re^{\ell}$, $z(t)\in\Re^{\ell}$, $K_0\in\Re^{\ell\times\ell}$, $K_1\in\Re^{\ell\times\ell}$, $g>0$, and $M\in\Re^{\ell\times(n-\ell)}$.

The closed-loop state and output equations of the system are, respectively,

$$\begin{bmatrix}\dot{z}(t)\\ \dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}0 & -F_1 & -F_2\\ 0 & A_{11} & A_{12}\\ gB_2K_1 & A_{21}-gB_2K_0F_1 & A_{22}-gB_2K_0F_2\end{bmatrix}$$

$$\times\begin{bmatrix}z(t)\\ x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}I_\ell\\ 0\\ gB_2K_0\end{bmatrix}v(t) \qquad (10)$$

and

$$y(t)=[0\;\; C_1\;\; C_2]\begin{bmatrix}z(t)\\ x_1(t)\\ x_2(t)\end{bmatrix} \qquad (11)$$

The transfer function matrix relating the plant output vector $y(t)$ to the command input vector $v(t)$ is obtained from Eqs. (10) and (11):

$$G(\lambda)=C_{c\ell}[\lambda I-A_{c\ell}]^{-1}B_{c\ell}=[0\;\; C_1\;\; C_2]$$

$$\times\begin{bmatrix}\lambda I_\ell & F_1 & F_2\\ 0 & \lambda I_{n-\ell}-A_{11} & -A_{12}\\ -gB_2K_1 & -A_{21}+gB_2K_0F_1 & \lambda I_\ell-A_{22}+gB_2K_0F_2\end{bmatrix}^{-1}$$

$$\times\begin{bmatrix}I_\ell\\ 0\\ gB_2K_0\end{bmatrix} \qquad (12)$$

By regarding $\epsilon=1/g$ as the perturbation parameter, the asymptotic form as $g\to\infty$ of the closed-loop transfer function matrix is[3]

$$\Gamma(\lambda)=\tilde{\Gamma}(\lambda)+\hat{\Gamma}(\lambda) \qquad (13)$$

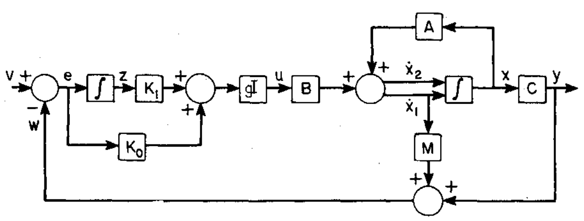

Fig. 1 Tracking system.

where

$$\tilde{\Gamma}(\lambda)=C_0(\lambda I_n-A_0)^{-1}B_0 \qquad (14)$$

$$\hat{\Gamma}(\lambda)=C_2(\lambda I_\ell-gA_4)^{-1}gB_2K_0 \qquad (15)$$

$$A_0=\begin{bmatrix}-K_0^{-1}K & 0\\ A_{12}F_2^{-1}K_0^{-1}K_1 & A_{11}-A_{12}F_2^{-1}F_1\end{bmatrix} \qquad (16)$$

$$B_0=\begin{bmatrix}0\\ A_{12}F_2^{-1}\end{bmatrix} \qquad (17)$$

$$C_0=[C_2F_2^{-1}K_0^{-1}K_1\quad C_1-C_2F_2^{-1}F_1] \qquad (18)$$

$$A_4=-B_2K_0F_2 \qquad (19)$$

In Eq. (13), $\tilde{\Gamma}$ and $\hat{\Gamma}$ are termed "slow" and "fast" asymptotic transfer functions since they contain only slow and fast modes, respectively. The slow modes of the closed-loop tracking system are those that are not direct functions of g. The corresponding poles of $\tilde{\Gamma}(\lambda)$ are easily found from

$$\det[\lambda I_n-A_0]=0 \qquad (20)$$

Since A_0 is block lower triangular, two sets of slow poles Z_1 and Z_2 are described, respectively, by

$$Z_1=\{\lambda\in\mathbb{C}: |\lambda K_0+K_1|=0\} \qquad (21)$$

and

$$Z_2=\{\lambda\in\mathbb{C}: |\lambda I_{n-\ell}-A_{11}+A_{12}F_2^{-1}F_1|=0\} \qquad (22)$$

The set Z_2 contains the transmission zeros of the system. The fast modes are direct functions of g. The corresponding poles of $\hat{\Gamma}(\lambda)$ are found from

$$\det[\lambda I_\ell-gA_4]=0 \qquad (23)$$

and comprise the set

$$Z_3=\{\lambda\in\mathbb{C}: |\lambda I_\ell+gF_2B_2K_0|=0\} \qquad (24)$$

It follows from Eqs. (14) and (16-18) that the slow transfer function is

$$\tilde{\Gamma}(\lambda)=(C_1-C_2F_2^{-1}F_1)(\lambda I_{n-\ell}-A_{11}+A_{12}F_2^{-1}F_1)^{-1}A_{12}F_2^{-1} \qquad (25)$$

Because of the block structure of the matrices in Eqs. (16) and (17), the slow modes containing the poles Z_1 become uncontrollable as $g\to\infty$. Thus, the transfer function $\tilde{\Gamma}(\lambda)$ in Eq. (25) contains only the slow modes associated with Z_2. Also, it follows from Eqs. (15) and (19) that the fast transfer function is

$$\hat{\Gamma}(\lambda)=C_2F_2^{-1}(\lambda I_\ell+gF_2B_2K_0)^{-1}gF_2B_2K_0 \qquad (26)$$

In order to achieve decoupling, it is necessary that both $\tilde{\Gamma}(\lambda)$ and $\hat{\Gamma}(\lambda)$ be diagonal. The conditions under which this is possible and the selection of the design parameters are presented in the remainder of this paper.

III. Synthesis for Decoupling

Design of the tracker requires the selection of the measurement matrix M and the control law matrices K_0 and K_1. In addition, achieving decoupling requires that the asymptotic transfer functions of Eqs. (25) and (26) must be diagonal. The conditions that must be satisfied are:

1) M must be chosen so that F_2 is not rank deficient and the matrix product F_2B_2 has full rank.

2) All of the closed-loop poles must lie in the open left half of the complex plane $\mathcal{C}$.

In order for the fat transfer function $\hat{\Gamma}(\lambda)$ of Eq. (26) to be diagonal it is necessary that

$$F_2B_2K_0 = (C_2 + MA_{12})B_2K_0 = \Sigma = \mathrm{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_\ell\} \quad (27)$$

where the selection is made so that all $\sigma_i > 0$ $(i = 1,2,\ldots,\ell)$. Then the fast eigenvalues are

$$Z_3 = \{-\sigma_1 g, -\sigma_2 g, \ldots, -\sigma_\ell g\} \quad (28)$$

A synthesis procedure for selecting the measurement matrix is to form $F_2 = C_2 + MA_{12}$ and to select the most sparse matrix M that produces a matrix F_2 of full rank. Simultaneously, the matrix product $C_2F_2^{-1}$ must be made diagonal. This procedure is straightforward and is illustrated in the examples in Sec. IV. It is not always possible to satisfy all of the necessary conditions to achieve a diagonal matrix $\Gamma(\lambda)$.

Diagonalizing the slow transfer function $\tilde{\Gamma}(\lambda)$ in Eq. (25) is considered by expanding it to the form

$$\tilde{\Gamma}(\lambda) = \Gamma_A(\lambda) - \Gamma_B(\lambda) \quad (29)$$

where

$$\Gamma_A(\lambda) = C_1(\lambda I - A_{11} + A_{12}F_2^{-1}F_1)^{-1}A_{12}F_2^{-1} \quad (30)$$

$$\Gamma_B(\lambda) = (C_2F_2^{-1})F_1(\lambda I - A_{11} + A_{12}F_2^{-1}F_1)^{-1}A_{12}F_2^{-1} \quad (31)$$

An equivalent block diagram representation of $\Gamma_A(\lambda)$ is shown in Fig. 2 and is based on

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 \quad (32)$$

$$\bar{y} = C_1x_1 \quad (33)$$

$$x_2 = -F_2^{-1}F_1x_1 + F_2^{-1}\bar{v} \quad (34)$$

Two design methods result from this representation.

Method 1

The block diagram of Fig. 2 fits the format of a class of systems for which the necessary and sufficient conditions for decoupling have been developed.[4] This is based on a system having the state, output, and control law given, respectively, by

$$\dot{x} = Ax + Bu \quad (35)$$

$$y = Cx \quad (36)$$

$$u = Fx + Gv \quad (37)$$

There is a pair of matrices F and G that decouples the system of Eqs. (35-37), provided that y and u have the same dimension, the number of outputs does not exceed the number of states, and

$$\det B^* = \det \begin{bmatrix} c_1^TA^{d_1}B \\ \vdots \\ c_m^TA^{d_m}B \end{bmatrix} \neq 0 \quad (38)$$

where m is the number of control inputs, c_i^T the ith row of C, and

$$d_i = \min\{j : c_i^TA^jB \neq 0, \quad j = 0,1,\ldots, \quad n-1\} \quad (39a)$$

or

$$d_i = n-1 \quad \text{if} \quad c_i^TA^jB = 0 \text{ for all } j \quad (39b)$$

The matrices F and G for the control law, Eq. (37), may be chosen[4] as

$$F = (B^*)^{-1}\sum_{k=0}^{\delta}(N_kCA^k - A^*) \quad (40)$$

$$G = (B^*)^{-1} \quad (41)$$

where $\delta = \max d_i$, the N_k are suitably chosen diagonal matrices

$$N_k = \mathrm{diag}\{n_k, \ldots, n_k^i, \ldots, n_k^m\}, \ i = 1,2,\ldots,m \quad (42)$$

and

$$A^* = \begin{bmatrix} c_1^TA^{d_1} \\ \vdots \\ c_m^TA^{d_m} \end{bmatrix} A \quad (43)$$

The block diagram in Fig. 2 represents Eqs. (35-37) provided that $A_{11} = A$, $A_{12} = B$, $C_1 = C$, $-F_2^{-1}F_1 = F$, and $F_2^{-1} = G$. In Ref. 4 the matrices F and G may be selected independently. However, in the tracker of this paper [Eqs. (3-6)], F and G are not independent since, in accordance with Eq. (5),

$$F = -F_2^{-1}F_1 = [C_2 + MA_{12}]^{-1}[C_1 + MA_{11}]$$
$$= (B^*)^{-1}[C_1 + MA_{11}] \quad (44)$$

$$G = F_2^{-1} = [C_2 + MA_{12}]^{-1} = (B^*)^{-1} \quad (45)$$

The appliation of this design method is illustrated in example 1 of Sec. IV.

Method 2

There is a class of systems for which the necessary conditions of method 1 are not satisfied. In those cases, when $\det|B^*| = 0$ and the number of outputs in Eq. (33) exceeds the number of states in Eq. (32), a new design method is developed in this paper and is illustrated in example 2 of Sec. IV.

As a summary, the selection of the proper design method depends on the properties of the system equations and is summarized in Fig. 3. Each class of system has a systematic

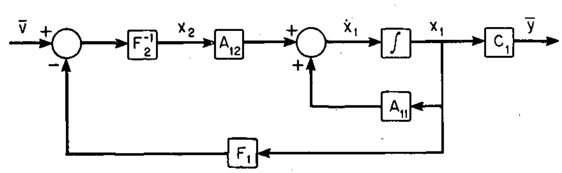

Fig. 2 Block diagram representing Eqs. (3) and (32-34).

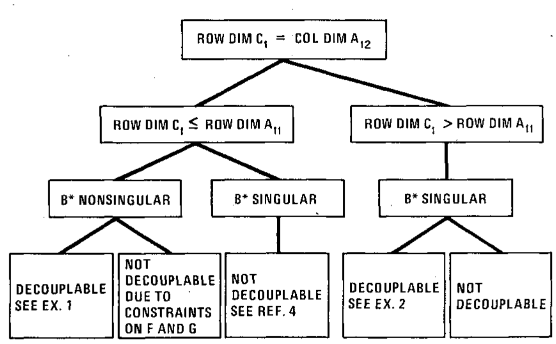

Fig. 3 Selection of design method.

procedure for obtaining a measurement matrix M that decouples the system. However, it should be noted that complete decoupling is not possible for all systems, even though they may satisfy the conditions of methods 1 or 2.

IV. Numerical Examples

Example 1

The linearized longitudinal equations of motion for an experimental V/STOL aircraft in a hovering condition are given in Ref. 4. By rearranging the order of the state variables, the state and output equations have the form given in Eqs. (3) and (4) and are shown as

$$\begin{bmatrix} \dot{\theta} \\ \Delta\dot{x} \\ \Delta\dot{z} \\ \dot{u} \\ \dot{q} \\ \dot{w} \end{bmatrix} = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \hline X_\theta & 0 & 0 & X_u & 0 & 0 \\ 0 & 0 & 0 & M_u & M_q & M_w \\ 0 & 0 & 0 & 0 & Z_q & Z_w \end{array}\right] \begin{bmatrix} \theta \\ \Delta x \\ \Delta z \\ u \\ q \\ w \end{bmatrix} + \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline X_{cv} & 0 & 0 \\ M_{cv} & M_{\theta nf} & 0 \\ 0 & Z_\theta & Z_{cs} \end{array}\right] \begin{bmatrix} \delta_v \\ \delta_{nf} \\ \delta_s \end{bmatrix} \tag{46}$$

$$\begin{bmatrix} \theta \\ \Delta x \\ \Delta z \end{bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} \theta \\ \Delta x \\ \Delta z \\ u \\ q \\ w \end{bmatrix} \tag{47}$$

u = incremental longitudinal (x) velocity change
θ = incremental pitch angle
q = pitch rate
w = incremental vertical (z) velocity change
δx = incremental position error
Δz = incremental altitude error
δ_v = incremental collective fan input
δ_{nf} = incremental nose fan input
δ_s = incremental fan stagger input

and where $A = A_{11} = 0$, $C = C_1 = I$, $C_2 = 0$, and

$$B = A_{12} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{48}$$

Applying Eqs. (39) and (38) yields $d_1 = d_2 = d_3 = 0$ and

$$B^* = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{49}$$

Since the row dimensions of C_1 = column dimension of A_{12} = dimension of A_{11} and since B^* is nonsingular, the conditions of method 1 are satisfied and only the constraints of Eqs. (44) and (45) remain to be investigated. Since $C_2 = 0$,

$$F_2 = C_2 + MA_{12} = MA_{12}$$

$$= \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 1 & m_{11} & 0.002m_{11} + m_{12} \\ 0 & m_{21} & 0.0026m_{21} + m_{22} \\ 0 & m_{31} & 0.0026m_{31} + m_{32} \end{bmatrix} \tag{50}$$

Comparing Eqs. (49) and (50) and modifying Eq. (45) in order to provide flexibility in the set of closed-loop poles Z_2 (transmission zeros) given by Eq. (22), the selection made for F_2 is

$$F_2 = \begin{bmatrix} 0 & m_{11} & 0 \\ m_{22} & 0 & 0 \\ 0 & 0 & m_{33} \end{bmatrix} \tag{51}$$

From Eq. (43), $A^* = 0$ since $A = A_{11} = 0$. Also, in Eq. (40), $\delta = 0$ and N_0 is a diagonal matrix. Thus, Eqs. (40) and (44) yield

$$F_1 = -N_0 = C_1 = I \tag{52}$$

which is diagonal for any measurement matrix M.

In the asymptotic transfer functions $\hat{\Gamma}(\lambda) = 0$ and $\Gamma_B(\lambda) = 0$ since $C_2 = 0$. Thus, $\Gamma(\lambda)$ is obtained from Eq. (30) as

$$\Gamma(\lambda) = \Gamma_A(\lambda) = \begin{bmatrix} \dfrac{1/m_{11}}{\lambda + 1/m_{11}} & 0 & 0 \\ 0 & \dfrac{1/m_{22}}{\lambda + 1/m_{22}} & 0 \\ 0 & 0 & \dfrac{1/m_{33}}{\lambda + 1/m_{33}} \end{bmatrix} \tag{53}$$

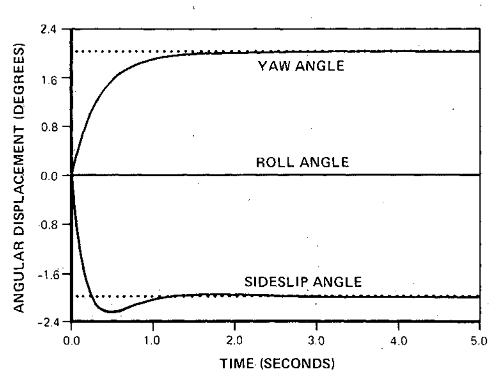

Fig. 4 Yaw pointing maneuvers.

Decoupling is achieved as shown in Eq. (53). The values of m_{11}, m_{22}, and m_{33} are selected to obtain the desired closed-loop tracking responses. Then Eqs. (27) and (21) are used to assign the required control law matrices K_0 and K_1.

Example 2

The linearized lateral-directional equations of motion of an aircraft[8] are arranged and partitioned in the form given in Eqs. (3) and (4),

$$\begin{bmatrix} \dot\phi \\ \dot\psi \\ \dot\beta \\ \dot p \\ \dot r \end{bmatrix} = \left[\begin{array}{cc|ccc} 0 & 0 & 0 & 1 & 0.002591 \\ 0 & 0 & 0 & 0 & 1 \\ \hline 0.0634136 & 0 & -0.428757 & 0.00281098 & -0.99507 \\ 0 & 0 & -12.0621 & -4.87384 & 1.09841 \\ 0 & 0 & 6.37457 & -0.202993 & -0.390207 \end{array}\right] \times \begin{bmatrix} \phi \\ \psi \\ \beta \\ p \\ r \end{bmatrix} + \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0.00123365 & 0.0069746 \\ 2.07524 & 0.0269759 & -0.00636638 \\ 0.0397057 & -0.0863931 & -0.00867571 \end{array}\right] \times \begin{bmatrix} \delta_w \\ \delta_r \\ \delta_{sf} \end{bmatrix} \quad (54)$$

$$\begin{bmatrix} \beta \\ \phi \\ \psi \end{bmatrix} = \left[\begin{array}{cc|ccc} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} \phi \\ \psi \\ \beta \\ p \\ r \end{bmatrix} \quad (55)$$

where

$$A = A_{11} = 0 \quad (56)$$

$$B = A_{12} = \begin{bmatrix} 0 & 1 & 0.002591 \\ 0 & 0 & 1 \end{bmatrix} \quad (57)$$

$$C = C_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (58)$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (59)$$

The conditions of method 1 are not satisfied since the row dimension of $C_1 >$ dimension of A_{11}. Equation (39) yields $d_1 = 1$ and $d_2 = d_3 = 0$. Forming B^* yields

$$B^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0.0026 \\ 0 & 0 & 1 \end{bmatrix} \quad (60)$$

with

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \\ m_{31} & m_{32} \end{bmatrix} \quad (61)$$

$$F_2 = C_2 + MA_{12} = \begin{bmatrix} 1 & m_{11} & 0.0026m_{11} + m_{12} \\ 0 & m_{21} & 0.0026m_{21} + m_{22} \\ 0 & m_{31} & 0.0026m_{31} + m_{32} \end{bmatrix} \quad (62)$$

The assignable elements of F_2 are permitted nonzero values only if B^* has a nonzero element in the corresponding position. The matrix F_2 must have full rank and the sparsest measurement matrix M is desired; thus

$$F_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & m_{21} & 0.0026m_{21} \\ 0 & 0 & m_{32} \end{bmatrix} \quad (63)$$

and all elements of m are zero except m_{21} and m_{32}. The matrix $C_2F_2^{-1} = C_2$ is diagonal, as required. Also, since $A_{11} = 0$, $F_1 = C_1 + MA_{11} = C_1$.

With the assignment $F_2B_2K_0 = \text{diag}\ \{\sigma_1, \sigma_2, \sigma_3\}$ in accordance with Eq. (27), the asymptotic closed-loop transfer function given by Eq. (14) yields

$$\Gamma(\lambda) = \tilde\Gamma(\lambda) + \hat\Gamma(\lambda)$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & \dfrac{1/m_{21}}{\lambda + 1/m_{21}} & 0 \\ 0 & 0 & \dfrac{1/m_{32}}{\lambda + 1/m_{32}} \end{bmatrix} + \begin{bmatrix} \dfrac{g\sigma_1}{\lambda + g\sigma_1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \dfrac{g\sigma_1}{\lambda + g\sigma_1} & 0 & 0 \\ 0 & \dfrac{1/m_{21}}{\lambda + 1/m_{21}} & 0 \\ 0 & 0 & \dfrac{1/m_{32}}{\lambda + 1/m_{32}} \end{bmatrix} \quad (64)$$

This transfer function has the desired diagonal form and thus decoupling is achieved. The desired closed-loop eigenvalues $\lambda_1 = -g\sigma_1$, $\lambda_2 = -1/m_{21}$, and $\lambda_3 = -1/m_{32}$ are selected by choosing the values σ_1, m_{21}, and m_{32} when computing the control law matrices in accordance with Eqs. (27) and (21). The complete design in accordance with Eqs. (27) and (21) is $\Sigma = \text{diag}\ \{0.05, 1, 1\}$, $g = 100$,

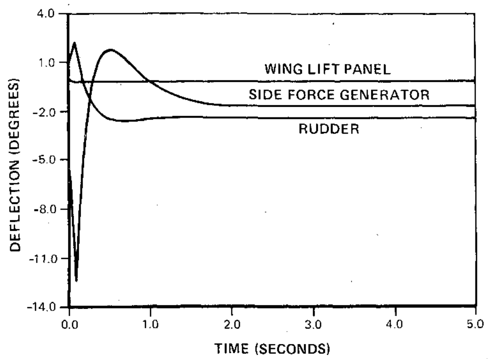

Fig. 5 Controls for yaw pointing.

$$M=\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1/3 \end{bmatrix}$$

and

$$K_0=K_1=\begin{bmatrix} 0.0317 & 4.7884 & 0.4720 \\ -0.7181 & 2.2405 & -35.1324 \\ 7.2959 & -0.3963 & 6.2141 \end{bmatrix} \tag{65}$$

A yaw pointing maneuver is commanded. This consists of a yaw angle command with the sideslip angle equal in magnitude and opposite in sign to the yaw angle. Also, roll angle is to be minimized. The response and controls are shown in Figs. 4 and 5. The aircraft responds rapidly with only 0.02 deg maximum roll angle perturbation. The desired fast response and decoupling are achieved. The control surface rates can be reduced by modification of the design, resulting in a slower response.

To study the robustness of the overall system, the uniform gain and phase margins are calculated[9,10] by breaking the loops at the inputs $\boldsymbol{u}$ in Fig. 1. The uniform gain and phase margins are found to be $(0.5,\infty)$ and ± 60 deg. These good characteristics are to be expected since this is a high-gain asymptotic design method. However, if noise is modeled, the system will not exhibit such high margins.

V. Conclusions

High-gain, error-actuated control is used to design a multivariable tracking system. The system compares a constant command vector with the output vector to generate an error vector. The input to the plant is proportional to both the error and the integral of error and steady-state tracking is achieved. For improper systems (the product of the output and input matrices is rank deficient), it is necessary to augment the system with a set of extra measurements. While this assures tracking, additional requirements are imposed to achieve decoupling. Transient decoupling requires that the closed-loop transfer function matrix be made diagonal. In this paper, the decoupling is accomplished by designing a transducer matrix such that the asymptotic structure of the closed-loop transfer matrix is diagonal. A synthesis technique for decoupling is developed for two classes of problems. The inherent feature of this technique is that, under the given tracking constraints, it generates the sparsest possible transducer matrix—which is required in practice. The method is illustrated through the design of control systems for an advanced fighter aircraft performing a direct force maneuver.

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